electric field on the anode* and j_{hi} is reduced to the value of $j_{hi} \sim j_{e0}(m/M_h)^{1/2}$. Correspondingly, the current of light ions is also reduced ($j_{li} \sim j_{0e}(m/M_h)^{1/2}/\alpha$) and this leads to the reduction of j_e to the initial value j_{e0}. Only as the diode gap is being filled by heavy ions and the electric charge is neutralized by them does j_e increase (the characteristic time now is τ_h) to the level corresponding to the stationary solution with ion flows.

The above considerations are illustrated by computation results for variants with $\alpha = 0.5; 0.8$. Since now $\alpha \sim 1$, $\Delta t \sim \tau_l$, the effects characteristic for the initial stage of the process appear in the diagrams as splashes j_e and j_{li} of duration $\sim \tau_l$. A further slow change of j_e (see Fig. 6) is due to the motion of heavy ions.

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*Here it may be essential to take into account the thermal expansion of the plasma. If the expansion rate is sufficiently high, then the plasma may cover a thin layer of heavy ions neutralizing their charge. Therefore, the described pattern takes place only when the anode plasma is sufficiently cool.

NUMERICAL SIMULATION OF THE SELF-FOCUSING OF WAVE PACKETS IN A MEDIUM WITH STRICTION NONLINEARITY

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During the propagation of powerful laser pulses in many media (e.g., crystals and plasma), the nonlinear increment to the dielectric constant associated with the development of sound perturbations may be very considerable. Striction nonlinearity may lead to the self-focusing of laser pulses, which in turn may be accompanied by the development of severe elastic stresses in crystals.

In this paper we shall make a numerical study of the propagation of axially symmetric wave packets in a medium with striction nonlinearity within the framework of the equations [1, 2]

$$i(u_t + vu_z) + \Delta_\perp u + \sigma\rho u = 0, \qquad (1)$$
$$\rho_{tt} - c_s^2\Delta\rho = -\Delta|u|^2$$

and the natural boundary conditions

$$\partial u/\partial r|_{r=0} = \partial\rho/\partial r|_{r=0} = 0,$$
$$u(r=\infty) = \rho(r=\infty) = 0,$$
$$u(|z|=\infty) = \rho(|z|=\infty) = 0,$$

where u is the envelope of the wave packet; v and c_s are the group velocity of light and the velocity of sound in

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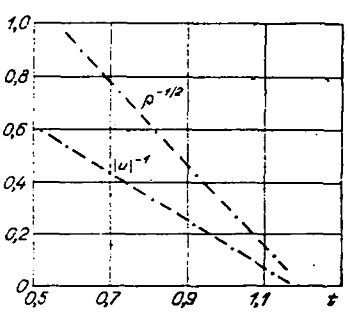

Fig. 1

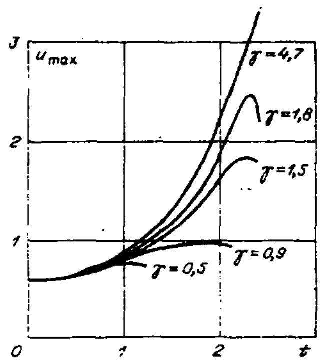

Fig. 2

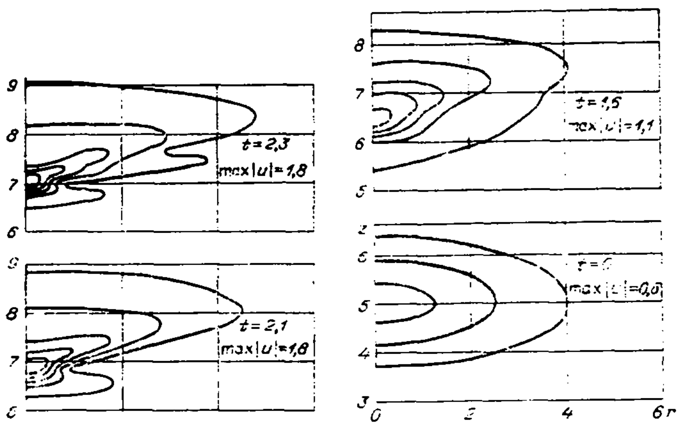

Fig. 3

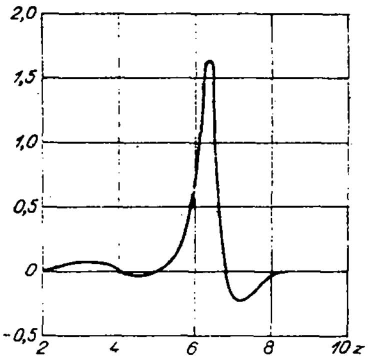

Fig. 4

the medium; ρ is the perturbation of the density of the medium; σ is a constant determined by the specific mechanism of excitation of the striction nonlinearity; $\Delta_\perp = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)$. All the quantities are made dimensionless in a corresponding manner.

In accordance with [3] the term describing the dispersion ($\sim u_{zz}$) is omitted from (1).

Specific numerical experiments were carried out for the initial conditions

$$u(t=0) = A\exp(-r^2/l^2 - (z-a)^2/L^2),$$
$$\rho(t=0) = |u(t=0)|^2/c_s^2, \quad \rho_t(t=0) = 0.$$

Let us first consider the propagation of long pulses L. In this case we may neglect the longitudinal derivatives in (1). We thus obtain

$$iu_t + \Delta_\perp u + \sigma\rho u = 0, \quad \rho_{tt} - c_s^2\Delta_\perp u = -\Delta |u|^2. \tag{2}$$

We may convince ourselves by direct substitution that as $t \to t_0$ the system (2) asymptotically satisfies an automodel substitution analogous to that given in [4]:

$$\rho = (\lambda^2/(t_0 - t)^2)R(\lambda r/(t_0 - t)), \ u = (\lambda/(t_0 - t))U(\lambda r/(t_0 - t)). \tag{3}$$

Since the solution (3) has a singularity at $t \to t_0$, this means that for extended pulses (beams) a collapse may be created in a finite time t_0. The possibility of the solution (2) passing out to the automodel state is confirmed by the numerical experiment. Figure 1 shows the maximum values of ρ and $|u|$ as functions of t calculated for a beam ($L = \infty$) with parameters $A = 1$; $l = 3$ at $\sigma = 5$.

Let us make some estimates regarding the conditions for the development of such a collapse and its time of development t_0. For this purpose we consider the stability of a stationary solution of the system (2) in the form

$$\rho = \rho_0, \ u = a_0 \exp(i\varphi),$$

where ρ_0, a_0 are constants; $\varphi = \sigma_0\rho_0 t$. Substituting $\rho = \rho_0 + \delta\rho$, $u = (u_0 + \delta a)\exp(i\varphi + i\delta\varphi)$ in (2), where $\delta\varphi$, δa, $\delta\rho$ are small increments, linearizing the equations obtained for these, and then assuming that these increments are proportional to $\exp(ikr - i\omega t)$, we obtain the equations

$$i\omega\delta a + k^2 a_0\delta\varphi = 0, \ k^2\delta a + i\omega a_0\delta\varphi - \sigma a_0\delta\varphi = 0,$$

$$-2k^2 a_0\delta a + (c_s^2 k^2 - \omega^2)\delta\rho = 0.$$

These equations lead to the dispersion equation

$$2\omega_\mp^2 = k^4 + c_s^2 k^2 \pm \left[(k^4 + k^2 c_s^2)^2 - 4(c_s^2 k^6 - 2\sigma a_0^2 k^4)\right]^{1/2}. \tag{4}$$

We see from (4) that the solution is unstable if

$$k^2 c_s^2 - 2\sigma a_0^2 < 0. \tag{5}$$

The quantity a_0 may be identified with the characteristic initial amplitude and k regarded as equal to $1/l$, where l is the characteristic initial transverse dimension of the beam. Then a_0^2/k^2 has the sense of the power (intensity) P. We may thus write down the approximate condition for the self-focusing of wave beams (packets) in a medium with striction nonlinearity:

$$P > c_s^2/2\sigma.$$

On satisfying this condition the quantity $1/\mathrm{Im}(\omega)$ constitutes an estimate for the development time of the collapse t_0. If $k \ll 1$, then

$$t_0 \sim \sqrt{2}\, l \Big/ \left(\sqrt{c_s^4 + 8\sigma a_0^2} - c_s^2\right)^{1/2}. \tag{6}$$

Numerical experiments confirm the estimates (5) and (6).

For a pulse of initial length L it is reasonable to assume that a collapse is formed if the time of nonlinear interaction of the pulse with the medium $t_1 = L/v(1 - c_s/v)$ is much greater than t_0:

$$\gamma \equiv t_1/t_0 \gg 1. \tag{7}$$

If condition (7) is not satisfied, we should expect that after the point of maximum contraction of the wave packet the latter will spread by virtue of diffraction and dispersion. In the geometrical optics approximation the time t_{max} to reach the maximum may be estimated as

$$t_{max} \sim |t_1 t_0/(t_1 - t_0)|.$$

Figure 2 illustrates precisely this relationship as observed in numerical experiments; it shows the behavior of $u_{max} = \max |u(0, z, t)|$ as a function of t for pulses with parameters $A = 0.6$; $l = 3$; $L = 1$; $c_s = 1$ at $\sigma = 5$, v being a variable parameter. The quantity a_0 in the expression for γ is put equal to $^1/_2 A$. It should be noted that the specific value of the coefficient (here 1/2) has little effect on the value of γ.

Numerical experiments were carried out in the following range of variation of the initial pulse parameters: $0.5 \leq A \leq 1$; $l = 3$; $L = 1, \infty$; $c_s = 1$; $1 \leq v \leq 4$ for $\sigma = 5$, and also for $v/c_s \gg 1$ (up to 10^2). The qualitative picture remained unchanged.

Figures 3 and 4 present a typical picture of the evolution of the initial pulse in the absence of a collapse. One notices the following features:

1. For fairly long times t, increasing oscillations of the field $|u|$ and perturbations of the density ρ arise. In the front of the pulse ρ becomes negative, and the field is "thrown" to the periphery.

2. At a later stage in the propagation of the density a second maximum arises; this first lags relative to the principal maximum, stops, and then moves in the opposite direction at a velocity c_s. The latter situation is a consequence of the fact that in this region the field $|u|$ is weak and the second of Eqs. (1) becomes purely a wave equation.

3. With the progress of time the velocity of the maximum-field region diminishes and the sharpness of the leading edge of the wave of density increases. This may lead to the development of severe elastic stresses in the medium. Such a situation is possible, for example, for the motion of a laser focus in a nonlinear medium, when the velocity of the focus approaches that of sound [5].

For the numerical experiments we used an implicit-difference scheme of the second order of accuracy in all the variables, with a nonuniform space lattice. A typical time step was approximately 0.01, while for $v/c_s \gg 1$ it fell to $5 \cdot 10^{-4}$. The spatial steps were about 0.01. The stability of the scheme for various values of the parameters was verified by a repeated calculation with altered steps.

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